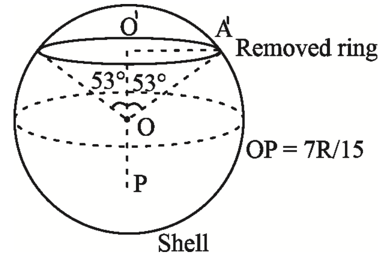
Solutions to JEE Advanced Home Practice Test -3 |JEE 2024 | Paper-1

PHYSICS

1.(C) Let surface charge density of shell equal σ.



$$\frac{kQ_{shell}}{R} = V_0$$

$$\frac{1}{4\pi\epsilon_0}\frac{\sigma 4\pi R^2}{R} = V_0$$

$$\frac{\sigma R}{\epsilon_0} = V_0 \Rightarrow \sigma = \frac{\epsilon_0 V_0}{R} \text{ ....(i)}$$

(A) Potential of centre after removal $= V_0 -$ (Potential due to ring at O)

$$= V_0 - \frac{k.Q_{ring}}{R} = V_0 - \frac{1}{4\pi\epsilon_0}\frac{\sigma.\alpha.4\pi R^2}{R}$$

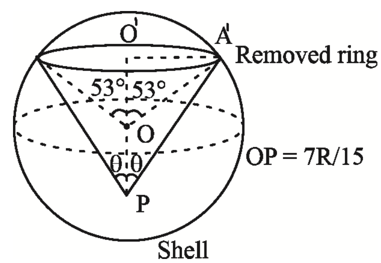
$$= V_0 - \frac{\sigma.\alpha.R}{\epsilon_0} = V_0 - \frac{\alpha R}{\epsilon_0}\frac{\epsilon_0 V_0}{R} = V_0 - \alpha V_0 = V_0(1-\alpha)$$

(B) Electric field at centre after removal = Electric field due to ring at centre along OO' direction

$$= \frac{kQ_{ring}\cos\theta}{R^2}\left(where\ \theta = 53°\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\alpha\sigma 4\pi R^2}{R^2}\cos 53° = \frac{3}{5}\frac{\alpha V_0}{R}$$

(C) Potential at point P $\left(OP = \frac{7R}{15}\right)$ after removal = Potential due to original shell – Potential due to ring at P



$$= V_0 - \frac{KQ_{ring}}{A'P} = V_0 - \frac{1}{4\pi\epsilon_0}\frac{\sigma\alpha 4\pi R^2\times 3}{4R}$$

$$= V_0 - \frac{3\alpha}{4}V_0 = V_0\left(1 - \frac{3\alpha}{4}\right)$$

$\rightarrow$ Let radius of ring $= r = R\sin 53° = \frac{4R}{5}$

$\rightarrow$ $O'O = R\cos 53° = \frac{3R}{5}$

$\rightarrow O'P = O'O + OP = \frac{3R}{5} + \frac{7R}{15} = \frac{9R+7R}{15} = \frac{16R}{15}$

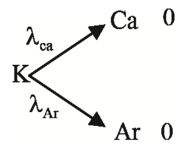
$\rightarrow \tan\theta = \frac{O'A'}{O'P} = \frac{15r}{16R} = \frac{15}{16R}.\frac{4R}{5} = \frac{3}{4} \Rightarrow \theta = 37°$

$\rightarrow (A'P)\sin\theta = O'A$ $\qquad\rightarrow$ $A'P = \frac{r}{\sin 37°} = \frac{4R\times 5}{5\times 3} = \frac{4R}{3}$

(D) Electric field at point P, after removal = electric field due to ring along PO', direction

$$= \frac{KQ_{ring}\cos\theta}{(A'P)^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\alpha 4\pi R^2}{\left(\frac{4R}{3}\right)^2}\cos 37° = \frac{9\alpha V_0}{20R}$$

2.(B) $t = 0$



$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2) \times N$$

$$\log_e\left(\frac{N}{N_0}\right) = -(\lambda_1 + \lambda_2)t$$

$$\because \frac{N_0 - N}{N} = 9 \Rightarrow N_0 - N = 9N \Rightarrow N = \frac{N_0}{10}$$

$$2.3\log_{10}\left(\frac{N_0/10}{N_0}\right) = -5 \times 10^{-10}t$$

$$2.3 \times -1 = -5 \times 10^{-10}t$$

$$t = \frac{2.3}{5} \times 10^{10}\, years = 0.46 \times 10^{10}\, years = 4.6 \times 10^{9}\, years$$

3.(A) $dQ = HdT$

$$\frac{dQ}{dt} = P = H\frac{dT}{dt} = HT_0 2\beta t$$

$$H.2\beta T_0 t = P$$

$$\because T = T_0\left(1 + \beta t^2\right) \Rightarrow 1 + \beta t^2 = \frac{T}{T_0} \Rightarrow t^2 = \left(\frac{T}{T_0} - 1\right)\frac{1}{\beta}$$

$$\Rightarrow t = \sqrt{\frac{T - T_0}{\beta T_0}}$$

So, $H = \dfrac{P}{2\beta T_0 t} = \dfrac{P}{2\beta T_0\sqrt{\dfrac{T - T_0}{\beta T_0}}} = \dfrac{P/2}{\sqrt{\beta T_0\left(T - T_0\right)}}$

4.(C) As per question, $L = m\omega r^2$

$\omega =$ Angular velocity at radius $(r) = \dfrac{L}{mr^2}$

Applying newtons Law,

$mg = m\omega^2 r$

$g =$ gravitational field at radial distance $(r) = \omega^2 r$

$$g = \frac{L^2}{m^2 r^3}$$

Work required to be done to go from $\left(r = \dfrac{R}{2}\right)$ to $(r = R)$

$$= \int_{r=R/2}^{r=R} Mg\,dr = \frac{ML^2}{m^2}\int_{R/2}^{R}\frac{1}{r^3}\,dr = \frac{ML^2}{2m^2}\left[-r^{-2}\right]_{R/2}^{R} = \frac{ML^2}{2m^2}\left[\frac{1}{r^2}\right]_{R}^{R/2}$$

$$= \frac{ML^2}{2m^2}\left[\frac{4}{R^2} - \frac{1}{R^2}\right] = \frac{3ML^2}{2m^2R^2}$$

5.(ABC)

$\left[mass\right] = M^0 L^0 T^0$

[Linear momentum] $= M^0 L^0 T^0$

Or, $\left[mass\right]\times\left[velocity\right] = M^0 L^0 T^0 \Rightarrow \left[velocity\right] = M^0 L^0 T^0$

$\because$ velocity $= \dfrac{Displacement}{time} = \left[velocity\right] = \dfrac{\left[Displacement\right]}{\left[time\right]} = \dfrac{L}{\left[time\right]} = M^0 L^0 T^0$

$\left[time\right] = L$

(A) $\left[acceleration\right] = \dfrac{\left[velocity\right]}{\left[time\right]} = \dfrac{M^0 L^0 T^0}{L} = L^{-1}$

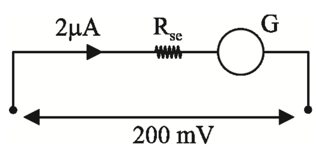
(B) $\left[volume\right] = \left[Length^3\right] = L^3$

(C) $\left[Volumetic\ flow rate\right] = \dfrac{\left[Volume\right]}{\left[time\right]} = \dfrac{L^3}{L} = L^2$

(D) $\left[velocity\right] = M^0 L^0 T^0$

6.(ABD)

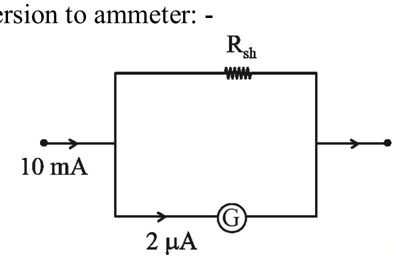
Conversion to voltmeter.



$$200\times10^{-3} = 2\times10^{-6}\left(R_{se}+10\right) \Rightarrow R_{se}+10 = 100\times10^3 = 100k\Omega$$

Resistance of voltmeter $= R_v = R_{se}+10 = 100k\Omega$.

Conversion to ammeter: -

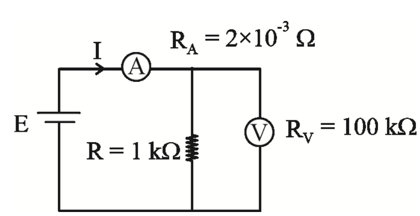


$$\Rightarrow 10\times2\times10^{-6} = \left(10\times10^{-3}-2\times10^{-6}\right)R_{sh}$$

$$\Rightarrow 20 = \left(10\times10^3-2\right)R_{sh} \ \Rightarrow R_{sh} \cong \frac{20}{10\times10^3} = 2\times10^{-3}\Omega$$

Resistance of Ammeter $= 2\times10^{-3}\Omega$

Circuit:



$$\because V = I.\frac{R.R_v}{R+R_v}$$

$$R_{measured} = \frac{V}{I} = \frac{RR_v}{R+R_v} = \frac{1\times100}{101}k\Omega \cong 990.1\Omega$$

Note : measured value is independent of nature of cell.

7.(ABCD) Electric field inside the wire at position $(x, y) = \vec{v} \times \vec{B} = -B_0\left(\frac{y}{L}\right)^{\beta} V_0 \hat{j}$

Small potential difference generated in length $'dl'$ in wire at location (x, y)

$$= dv = -\vec{E}.\vec{dl} = -\vec{E}.\left(dx\hat{i} + dy\hat{j}\right) = B_0V_0\left(\frac{y}{L}\right)^{\beta} dy$$

(A) If $\beta = 1$, then p.d across the wire $= \int dv = \int\limits_{y=0}^{L} B_0V_0\frac{y}{L}dy = \frac{B_0V_0}{L}\frac{L^2}{2} = \frac{B_0V_0L}{2}$

(B) If $\beta = 2$, then p.d across the wire $= \int dv = \int\limits_{y=0}^{L} \frac{B_0V_0y^2dy}{L^2} = \frac{B_0v_0}{L^2}\frac{L^3}{3} = \frac{B_0V_0L}{3}$

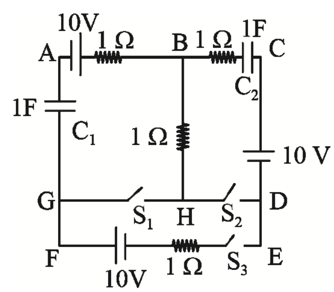
(C) If $\beta = 0$, then p.d. across the wire $= \int dv = \int\limits_{y=0}^{L} B_0V_0dy = B_0V_0L$

(D) If $\beta = \frac{1}{2}$, then p.d. across the wire

$$= \int dv = \int\limits_{y=0}^{L} B_0V_0\left(\frac{y}{L}\right)^{1/2} dy = \frac{B_0V_0}{\sqrt{L}}\left[\frac{2y^{3/2}}{3}\right]_0^L = \frac{2B_0V_0L}{3}$$

8.(ABD)

(A)



At $t = 0, s_1$ is closed

P.d. across $c_1 = 0$ in loop ABHGA, current via $c_1 = \frac{10}{2} = 5A$

(B) just after s_1 is closed, p.d. across s_2 :

In loop HBCDH,

$\because \quad V_H + 5 - 0 - 10 = V_D \Rightarrow V_H - 5 = V_D \Rightarrow V_H - V_D = 5 \Rightarrow V_{s_2} = 5\,volts$

(C) and (D)

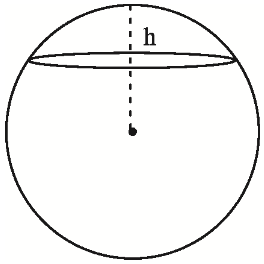
When s_1, s_2 *and* s_3 are closed, at steady state:

$i_{s_1} = i_{s_2} = i_{s_3} == 10A(loop\,GHDEFG)$

$Q_{C_1} = cv = 1 \times 10 = 10c$

$Q_{C_2} = cv = 1 \times 10 = 10c$

9.(ABCD)

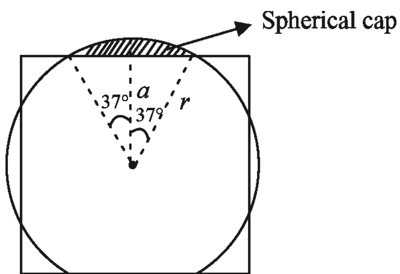


$r =$ radius of sphere

$h =$ height of spherical cap

Volume of spherical cap $= \dfrac{\pi h^2}{3}(3r - h)$

(A)



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$\because r\cos 37° = a$

$r.\dfrac{4}{5} = a \Rightarrow r = \dfrac{5a}{4}$

h = height of spherical cap = $r - a = \dfrac{5a}{4} - a = \dfrac{a}{4}$

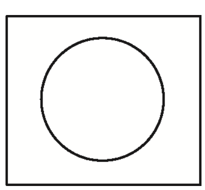
Volume of spherical cap $= \dfrac{\pi h^2}{3}(3r - h) = \dfrac{\pi}{3}\dfrac{a^2}{16}\left(3\times\dfrac{5a}{4} - \dfrac{a}{4}\right) = \dfrac{\pi a^2}{48}\times\dfrac{14a}{4} = \dfrac{7}{96}\pi a^3$

Volume enclosed in Gaussian surface = Volume of sphere - 6 $\times$ volume of spherical cap

$= \dfrac{4}{3}\pi r^3 - 6\times\dfrac{7}{96}\pi a^3 = \dfrac{4}{3}\pi\left(\dfrac{5a}{4}\right)^3 - \dfrac{42}{96}\pi a^3 = \dfrac{13}{6}\pi a^3$

$flux = \left(\dfrac{\rho 13}{\in_0 6}\pi a^3\right)$

(B)

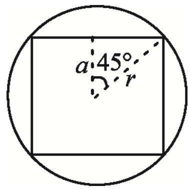


$r = \dfrac{a}{2}$

(Volume enclosed in Gaussian surface) $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi\dfrac{a^3}{8} = \dfrac{\pi a^3}{6}$

Flux $= \dfrac{\rho\pi a^3}{6\in_0}$

(C)



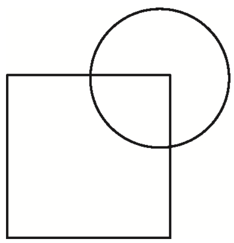
$r.\cos 45° = a$

$r = \sqrt{2}a$

Volume enclosed in Gaussian surface $(2a)^3 = 8a^3$

Flux $= \dfrac{8\rho a^3}{\epsilon_0}$

(D)



$\because r = a$

Volume enclosed in Gausian surface $= \dfrac{1}{8} \times \dfrac{4}{3}\pi r^3 = \dfrac{\pi}{6}a^3$

Flux $= \dfrac{\rho\pi a^3}{6\epsilon_0}$

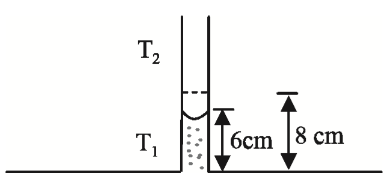
10.(AD) $r = 0.2 \times 10^{-3}\,m, T = 0.075\,N/m, \rho = 1000\,kg/m^3, \theta_1 = 37°, \theta_2 = 53°$

Height raised if T_1 is used of sufficient length

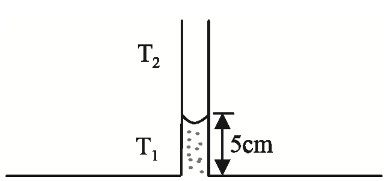
$= H_1 = \dfrac{2T\cos 37°}{\rho g r} = \dfrac{2 \times 0.075 \times 4}{5 \times 10^3 \times 10 \times 0.2 \times 10^{-3}} = 0.06\,m = 6\,cm$

Height raised if T_2 is used of sufficient length $= H_2 = \dfrac{2T\cos 53°}{\rho g r} = 4.5\,cm$

(i)



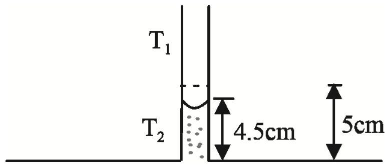
(ii)



The liquid will rise only upto 5 cm, then will adjust its radius of meniscus to compensate for pressure difference.

(iii) As contact angles are different, weight of water in meniscus can't be same in two tubes.

(iv)



11.(ABCD)

$*\ \delta_0 = (n-1)\theta$

$*\ \delta_1 = (n_1 - 1)\frac{\theta}{2};\ \delta_2 = (n_2 - 1)\frac{\theta}{2};$

$\Rightarrow \delta = (n_1 + n_2 - 2)\frac{\theta}{2}$

(A) If $n_1 = n - \Delta n, n_2 = n + \Delta n,$ then

$$\delta = \delta_0 + \Delta\delta = (n - \Delta n + n + \Delta n - 2)\frac{\theta}{2} = (2n-2)\frac{\theta}{2} = (n-1)\theta$$

$$\Delta\delta = 0$$

(B) If $n_1 = n + \Delta n, n_2 = n$, then

$$\delta_0 + \Delta\delta = (2n + \Delta n - 2)\frac{\theta}{2} = (n-1)\theta + \Delta n.\frac{\theta}{2}$$

$$\Delta\delta = \Delta n.\frac{\theta}{2}$$

For $n = 1.5, \Delta n = 10^{-3}, \theta = 1°, \Delta\delta = 10^{-3} \times \frac{1}{2} = 0.5 \times 10^{-3}\ \deg rees.$

(C) $\frac{\Delta\delta}{\delta_0} = \frac{(n_1 + n_2 - 2)\theta/2}{(n-1)\theta} = \frac{n_1 + n_2 - 2}{2(n-1)}$

$$n_1 = n; n_2 = n + \Delta n \Rightarrow \frac{\Delta\delta}{\delta_0} = \frac{2n + \Delta n - 2}{2(n-1)} = \frac{2(n-1) + \Delta n}{2(n-1)} = 1 + \frac{\Delta n}{2(n-1)}$$

$$\Rightarrow \frac{\Delta\delta}{\delta_0} = 1 + \frac{\Delta n}{2(n-1)}$$

$$\because n = 1.5, \Delta n = 10^{-3} \Rightarrow \frac{\Delta\delta}{\delta_0} = 1 + \frac{10^{-3}}{2 \times 0.5} = 1.001$$

(D) $n_1 = n, n_2 = n - \Delta n,$ then $\frac{\Delta\delta}{\delta_0} = 1 - \frac{\Delta n}{2(n-1)} = 1 - \frac{10^{-3}}{2 \times 0.5} = 0.999$

12. (ABD) **Process 1-2**

$$P = \frac{P_0}{T_0}(T - T_0) + P_0 = \frac{P_0}{T_0}.T$$

Process is iso-choric

$$W_{1\to 2} = \int_{v_1}^{v_2} PdV = 0$$

$$\Delta U_{1-2} = nC_v\Delta T = 1 \times \frac{3}{2}R \times T_0 = \frac{3}{2}RT_0$$

$$\Delta Q_{1-2} = \frac{3}{2} R \times T_0$$

Process $2 \rightarrow 3$ (iso-baric)

$$W_{2\rightarrow 3} = nR\Delta T = 1 \times R \times \left(-T_0\right) = -RT_0$$

$$\Delta U_{2\rightarrow 3} = nC_v \Delta T = 1 \times \frac{3R}{2} \times \left(-T_0\right) = -\frac{3}{2} RT_0$$

$$\Delta Q_{2\rightarrow 3} = -\frac{5RT_0}{2}$$

. Process $3 \rightarrow 4$: (iso-choric)

$$P = \frac{2P_0}{T_0}\left(T - \frac{T_0}{2}\right) + P_0 = \frac{2P_0}{T_0} \times T - P_0 + P_0$$

$$P = \frac{2P_0}{T_0} T$$

$$w_{3\rightarrow 4} = 0$$

$$\Delta U_{3\rightarrow 4} = nC_v \Delta T = 1 \times \frac{3R}{2} \times \frac{-T_0}{2} = -\frac{3RT_0}{4}$$

$$\Delta Q_{3\rightarrow 4} = -\frac{3RT_0}{4}$$

Process ($4 \rightarrow 1$): Isobaric

$$W_{4\rightarrow 1} = nR\Delta T = 1 \times R \times \frac{T_0}{2} = \frac{RT_0}{2}$$

$$\Delta U_{4\rightarrow 1} = nC_v \Delta T = 1 \times \frac{3R}{2} \times \frac{T_0}{2} = \frac{3RT_0}{4}$$

$$\Delta Q_{4\rightarrow 1} = \frac{5R}{4} T_0$$

(A) $w_{cyclic} = \sum w_{individual} = 0 - RT_0 + 0 + \frac{RT_0}{2} = -\frac{RT_0}{2}$

(B) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{2-3}}\right| = \frac{3/2}{5/2} = \frac{3}{5}$

(C) Only iso-baric and iso-choric

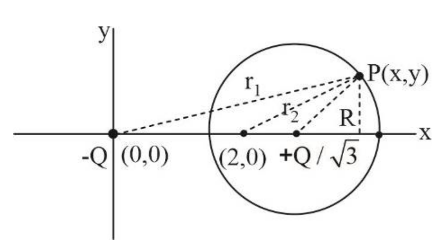
(D) $\left|\frac{\Delta Q_{3-4}}{\Delta Q_{4-1}}\right| = \frac{3/4}{5/4} = 0.6$

13.(1.73)

Let a point P on circle

$$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{kQ/\sqrt{3}}{r_2}$$

$$\frac{kQ}{r_1} = \frac{kQ/\sqrt{3}}{r_2}$$



$$\frac{1}{\sqrt{x^2 + y^2}} = \frac{1}{\sqrt{3}\sqrt{(x-2)^2 + y^2}}$$

$$3(x-2)^2 + 3y^2 = x^2 + y^2$$

$$3(x^2+4-4x)-x^2+2y^2=0$$
$$2x^2+12-12x+2y^2=0$$
$$x^2+6-6x+y^2=0$$
$$(x-3)^2+y^2=\left(\sqrt{3}\right)^2$$
$$R=\sqrt{3}=1.73=b$$
$$a=3+1.73=4.73$$

14. (21.00)

Applying $d_{app}=\dfrac{d}{\mu}$

$$d=(\mu)d_{app}$$
$$d_1=(1.5)(6)=9\,cm$$
$$d_2=(1.5)(8)=12\,cm$$

Therefore, actual thickness of the glass slab is $d_1+d_2=21cm$.

15.(9) $\sigma AT^4=-ms\dfrac{dT}{dt}$

$$\int_{200}^{100}\frac{dT}{T^4}=\int_0^{t_1}kdt$$
$$\left.\frac{1}{3T^3}\right|_{200}^{100}=kt_1$$
$$\frac{1}{3}\left(\frac{1}{100^3}-\frac{1}{200^3}\right)=kt_1$$
$$\left.\frac{1}{3T^3}\right|_{200}^{50}=kt_2$$
$$\frac{1}{3}\left(\frac{1}{50^3}-\frac{1}{200^3}\right)=kt_2$$
$$\frac{t_2}{t_1}=\left(\frac{200^3-50^3}{200^3-100^3}\right)\frac{100^3}{50^3}=9$$

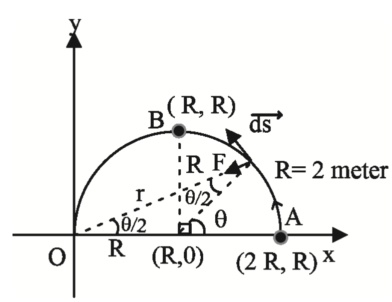
16. (1.38 - 1.39)

$$F=\frac{4}{r}$$

At some intermediate place:

$$r=2R\cos\frac{\theta}{2}$$

Magnitude of force $=\dfrac{4}{r}=\dfrac{4}{2R\cos\left(\dfrac{\theta}{2}\right)}$



Displacement in small time $=Rd\theta$ (tangential to circular path) angle between ds and force $=90°-\dfrac{\theta}{2}$

Small work done $= \vec{F}.\overrightarrow{ds} = \dfrac{4}{2R\cos\left(\dfrac{\theta}{2}\right)}.Rd\theta\cos\left(90-\dfrac{\theta}{2}\right),$

$$dw = 2\tan\left(\frac{\theta}{2}\right)d\theta$$

$$w = 2\int_{\theta=0}^{\pi/2}\tan\left(\frac{\theta}{2}\right)d\theta = 2\times 2\left\{\log_e\left[\sec\frac{\theta}{2}\right]\right\}_{\theta=0}^{\pi/2} = 4\log_e\left(\sqrt{2}\right) = 2\log_e 2 = 2\times 0.693 \cong 1.39$$

17.(1000)

Path difference, $\Delta x = S_1P - S_2P = \sqrt{(2)^2+(4)^2} - \sqrt{(1)^2+(4)^2} = 4.47-4.12 = 0.35\,m$

Constructive interference occurs when the path difference in an integer multiple of wavelength.

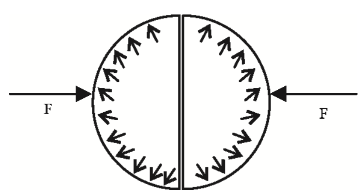
Or $\Delta x = n\lambda = \dfrac{nv}{f}$ $or\ f = \dfrac{nv}{\Delta x}$ where, $n = 1,2,3,......$

$$f = \frac{350}{0.35},\frac{2\times 350}{0.35},\frac{3\times 350}{0.35},.....$$

$f = 1000Hz, 2000Hz, 3000Hz, ...$ etc.

$$f = 1000n \Rightarrow f_0 = 1000Hz$$

18. (7.85) $\dfrac{\Delta V}{V} = Volumetic\, strain = \gamma\theta$



$B =$ Bulk modulus $= \dfrac{\delta P}{\dfrac{\Delta v}{v}}$

$$\Rightarrow \delta P = B.\frac{\Delta v}{v} = B\gamma\theta$$

To hold the half shell, minimum force required $= \delta P.\pi R^2 = B\gamma\theta.\pi R^2$

$R = 10\,cm = 0.1\,m,\ \theta = 5°C, \gamma = 10^{-6}, B = 5\times 10^7\, N/m^2$

$$F_{\min} = 5\times 10^7 \times 10^{-6}\times 5\times\pi\times 10^{-2}\,N = 78.50\times 10^{-1}\,N = 7.85N$$

CHEMISTRY

19.(B) $K_b = r > p$ (due to Guanidine derivative)

Lone pair of q (Nitrogen) involve in resonance with both oxygens.

20.(D) (A) Cassiterite is a tin oxide mineral SnO_2.

(B) Purified ore that contains 70% SnO_2 is called black tin.

$SnO_2 + 2C \rightarrow Sn + 2CO$ (carbon reduction)

(C) Roasting is done at moderate temperature in presence of air in reverberatory furnace. The air supply is stopped and temperature is increased to melt the mass when self-reduction takes place.

(D) At 500-800 K (lower temperature range in the upper part of blast furnace).

$3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$

$Fe_3O_4 + 4CO \longrightarrow 3Fe + 4CO_2$

$Fe_2O_3 + CO \longrightarrow 2FeO + CO_2$

At 900-1500 K (higher temperature range in the lower part of blast furnace):

$C + CO_2 \longrightarrow 2CO;\ FeO + CO \rightarrow Fe + CO_2$

21.(D) $Fe^{2+} + 2K^+ + \left[Fe(CN)_6\right]^{4-} \rightarrow K_2Fe\left[Fe(CN)_6\right]\downarrow$ [white] (in absence of air)

$Fe^{3+} + \left[Fe(CN)_6\right]^{4-} \rightarrow Fe_4\left[Fe(CN)_6\right]_3$ (Prussian blue precipitate or solution)

$Zn^{+2} + 2K^+ + \left[Fe(CN)_6\right]^{4-}$ (excess) $\rightarrow K_2Zn_3\left[Fe(CN)_6\right]_2\downarrow$ [bluish-white/white]

$Ca^{2+} + 2K^+ + \left[Fe(CN)_6\right]^{4-} \rightarrow K_2Ca\left[Fe(CN)_6\right]\downarrow$ [white]

$Cu^{+2} + \left[Fe(CN)_6\right]^{4-} \rightarrow Cu_2\left[Fe(CN)_6\right]\downarrow$ [chocolate brown]

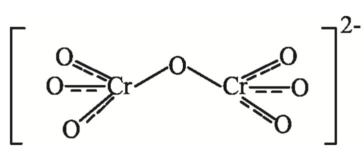
22.(A). On increasing concentration of $\left[H^+\right]$ ions the solubility of basic hydroxide $Fe(OH)_2$, will increase.

.

23.(ABCD) (A) $2Ag^+ + S_2O_8^{2-} \rightarrow 2Ag^{2+} + 2SO_4^{2-};\ Ag^{2+} + 4py \rightarrow \left[Ag(py)_4\right]^{2+}$ (red).

(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$.

(C)



(D) The colour of the complex ion is attributed to the presence of unpaired electron which is/are able to under go d-d transition in visible light. Ti^{2+} has two unpaired electrons.

So, d-d transition electron is possible while Ti^{4+} has no electrons in the d-orbitals.

24.(ABCD) Anomers have different configuration at glycosidic carbon.

25.(AC)

(B)

$$HA_{(g)} \longrightarrow HA_{(aq)} \qquad \Delta H = -70.7kJ$$

$$HA_{(aq)} \longrightarrow H^+_{(aq)} + A^-_{(aq)} \qquad \Delta H = 15kJ$$

$$BOH_{(aq)} \longrightarrow B^+_{(aq)} + OH^- \qquad \Delta H \simeq 0$$

$$H^+_{(aq)} + OH^-_{(aq)} \longrightarrow H_2O \qquad \Delta H = -57.3kJ$$

$$B^+_{(aq)} + A^-_{(aq)} \longrightarrow AB_{(aq)} \qquad \Delta H \simeq 0$$

$$HA_{(g)} + BOH_{(aq)} \longrightarrow BA_{(aq)} + H_2O \qquad \Delta_r H = -113kJ$$

(D) $BOH(g) + \text{aqueous} \rightarrow B^+(aq) + OH^-(aq) \qquad \Delta H = -20\,kJ/mole$

26.(A)

(A) $\dfrac{T_c}{P_C} = \dfrac{\frac{8a}{27Rb}}{\frac{a}{27b^2}} = \dfrac{8b}{R}$

(B) $T_b = \dfrac{a}{Rb},\ T_c = \dfrac{8a}{27Rb},\ T_c = \dfrac{8}{27}T_b,\ T_c < T_b$

(C) At critical point, behaviour of real gas is not ideal behaviour

27.(BCD) Compound $[X]$ is $Na_2B_4O_7$

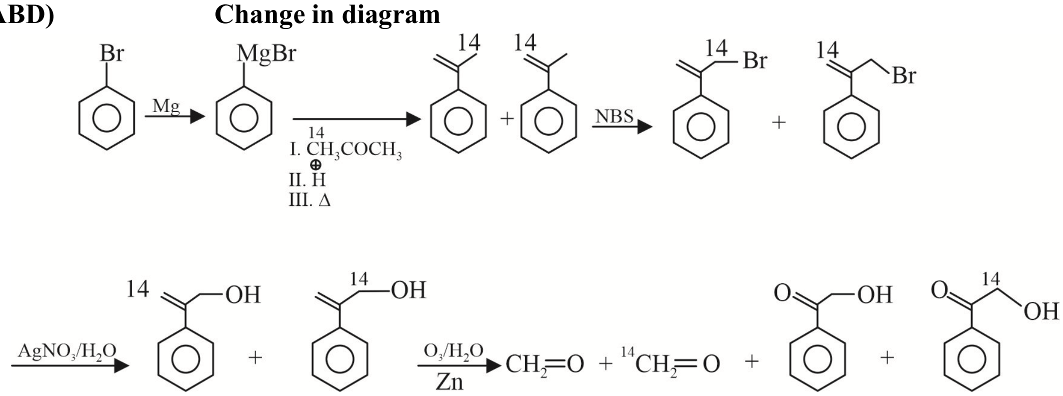
(A) Two boron atoms are sp^2 while other two boron atoms are sp^3 hybridized

(B) Product is sodium peroxoborate which is used as brightner in soaps.

(C) H_3BO_3 is formed which with HF gives HBF_4.

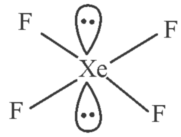
(D) Green bead in oxidizing as well as in reducing flame in cold.

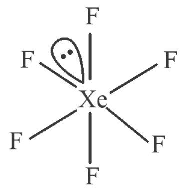
28.(ABD) **Change in diagram**

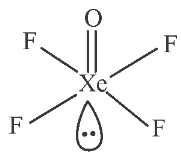


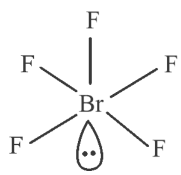
29.(BCD)

Polarity depend on net dipole moment. If dipole moment $\mu = 0$, it is nonpolar.

A.  XeF_4 $\mu=0$ non polar

B.  XeF_6 $\mu\neq0$ polar

C.  XeF_4 $\mu\neq0$ polar

D.  $\mu\neq0$ polar

30.(ACD) **(A)** $_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{140} + 2\,{}_0n^1 + {}^A_ZX$

Atomic no. $\Rightarrow 92+0 = 56+(2\times0)+Z,\ Z=36$

Mass no. $235+1=140+(2\times1)+A$

$A = 236-172 = 94$; So X is ${}^{94}_{36}Kr$

(B) $_{11}Na^{23} + {}^A_ZX \rightarrow {}_{12}Mg^{23} + {}_0n^1$

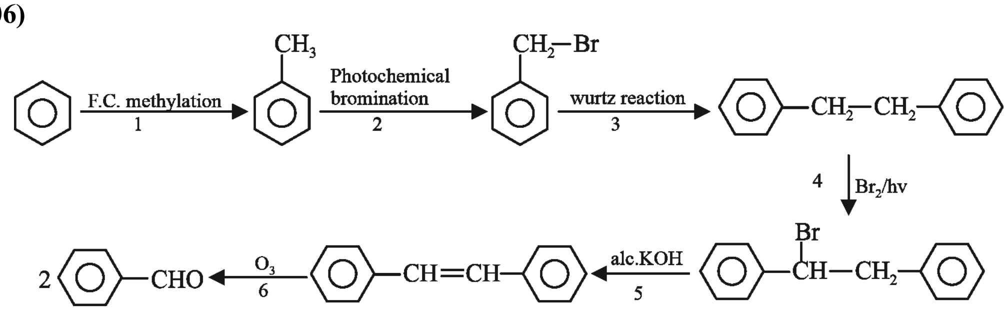
Atomic no. $\Rightarrow 11+Z=12+0 \Rightarrow Z=1$

Mass no. $\Rightarrow 23+A=23+1 \Rightarrow A=1$

$X \rightarrow {}^1_1H \rightarrow \text{Proton}$; Both (C) and (D) are correct.

31.(6) $NCl_3, NBr_3, BCl_3, PCl_3, PCl_5, SiCl_4$ are significantly hydrolysed under the normal conditions

32. (106)



(P)

M.F.(P) $= C_7H_6O$; M.W.$(P) = 84+6+16 = 106$

33.(12) $K_f = \dfrac{RT_f^2 M}{1000\,\Delta H_{fusion}} \left(M = \text{mol.wt.}\ ;\ T_f = \text{freezing piont of pure solvent}\right)$

$$K_f = \frac{8.314 \times (273)^2 \times 18}{1000 \times 6 \times 10^3}$$

$$\therefore\ K_f \approx 1.86\,\text{K.kg mol}^{-1}$$

Now $\Delta T_f = K_f \times m\ \ (m = \text{molality})$

$$m = \frac{\Delta T_f}{K_f} = \frac{273 - 271}{1.86} = 1.075\,\text{moles / kg}$$

But $m = \dfrac{\text{moles of solutes}}{\text{weight of solvent (in kg)}}$

$$n = \text{moles of solute} = 1.075 \times \left(\frac{0.9 \times 18}{1000}\right) = 1.74 \times 10^{-2}$$

Also $x_{solute} = \dfrac{760 - 700}{760} = 0.079$ (where x = mole fraction)

Total moles $= \dfrac{1.74 \times 10^{-2}}{0.079} = 0.22$; Moles of solvent $(H_2O) = 0.2026$

Mass of ice separated out $= (0.9 - 0.2026) \times 18 = 12.55\text{gm}$

34.(80)

$$Mg(OH)_2 \rightleftharpoons Mg^{+2} + 2OH^-$$

$x \quad 2x + 3y$ $\qquad$ K_{sp} of $Mg(OH)_2 > K_{sp}$ of $Al(OH)_3$

$$Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^- \qquad \text{so } x >> y$$

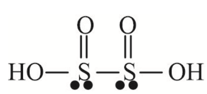
$y \quad 3y + 2x$ $\qquad$ $2x + 3y \simeq 2x$

$$4 \times 10^{-12} = \left[Mg^{2+}\right]\left[OH^-\right]^2 = x \times (2x)^2 = 10^{-4},\ x = 10^{-4}$$

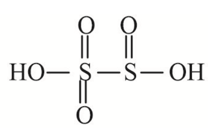
$$1 \times 10^{-33} = \left[Al^{3+}\right]\left[OH^-\right]^3 = (y)(2x)^3; \qquad 1 \times 10^{-33} = (y) \times \left(10^{-4} \times 2\right)^3$$

$$y = \frac{10^{-21}}{8}\ \text{so}\ \frac{x}{y} = 8 \times 10^{17};\ 8 \times 10^{17} \times 10^{-16} = 80$$

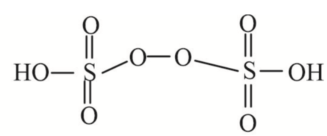
35.(4)



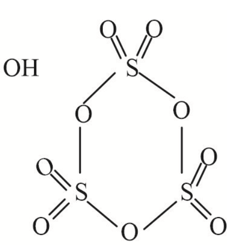
$(H_2S_2O_4)$



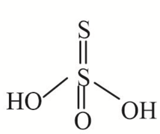
$(H_2S_2O_5)$



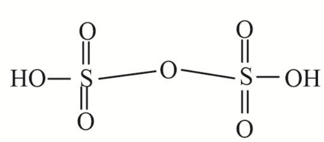
$(H_2S_2O_8)$



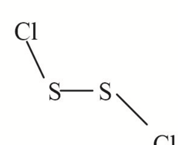
cyclic $(SO_3)_3$



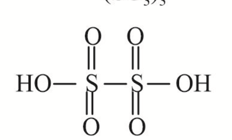
$(H_2S_2O_3)$



$(H_2S_2O_7)$



(S_2Cl_2)



$(H_2S_2O_6)$

36. (1.73)

$r_1 = k[0.01]^a [0.01]^b = 6.93 \times 10^{-6}$ ……..(i)

$r_2 = k[0.02]^a [0.01]^b = 1.386 \times 10^{-6}$ ………(ii)

$r_3 = k[0.02]^a [0.02]^b = 1.386 \times 10^{-6}$ ……..(iii)

$(0.02)^a = 2 \Rightarrow a = 1$

$(0.02)^b = 1 \Rightarrow b = 0$

$\Rightarrow k = 6.93 \times 10^{-4}\ \text{sec}^{-1}$

$A + 2B \rightarrow C$

$0.5 \quad 1 \rightarrow$ overall order $= 1$; $\qquad 6.93 \times 10^{-4} = \dfrac{3}{50 \times 60} \ln \dfrac{A_0}{A_t}$

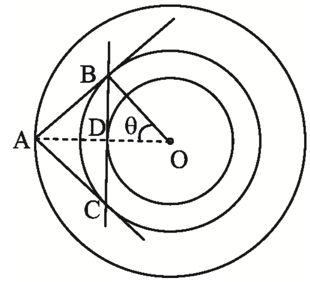
$0.693 = \ln \dfrac{A_0}{A_t} \Rightarrow \dfrac{A_0}{A_t} = 2 \Rightarrow A_t = 0.25; \Rightarrow \text{rate} = 6.93 \times 10^{-4} \times \left(\dfrac{1}{4}\right)$

Rate $= 1.73 \times 10^{-4}\,\text{M}\,\text{sec}^{-1}$

$\therefore\ 1.73 \times 10^{-4} \times 10^{4} = 1.73$

MATHEMATICS

37.(A)



$$\cos\theta = \frac{OD}{OB} = \frac{OB}{OA}$$

$$\frac{1}{r} = \frac{r}{9}$$

$$\Rightarrow r = 3$$

38.(B) $\because \int_0^1 \left(xf(x) - \frac{\sqrt{x}}{2} \right)^2 dx = 0$

$$\therefore\ f(x) = \frac{1}{2\sqrt{x}}$$

Number of solution is 1

39.(C) Let $x+y+z=\theta$ and $k=2$

$\therefore \cos x + \cos y + \cos z = k\cos\theta$ and $\sin x + \sin y + \sin z = k\sin\theta$

$$\cos(x+y) + \cos(y+z) + \cos(z+x) = \cos(\theta - z) + \cos(\theta - x) + \cos(\theta - y)$$

$$= \cos\theta(k\cos\theta) + \sin\theta(k\sin\theta) = k = 2$$

40.(A) $\because |z+16|^2 = 16|z+1|^2$

$$z\overline{z} + 16z + 16\overline{z} + 256 = 16(z\overline{z} + z + \overline{z} + 1)$$

$$15z\overline{z} = 240 \Rightarrow z\overline{z} = 16 \Rightarrow |z| = 4$$

41.(ABC) Subtracting the given two equations.

$5x^2 - 15x + b + a = 0$ has 2 roots α and β

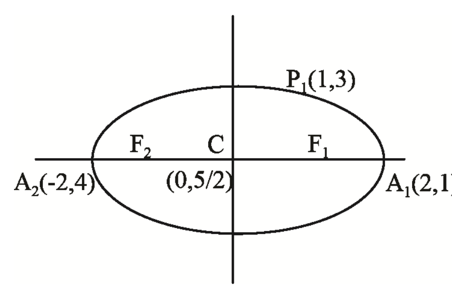
$\alpha + \beta = 3$; $\quad \alpha + \beta + x_1 = 5$

$\alpha + \beta + x_2 = 0$; $x_1 = 2$; $x_2 = -3$

Put x_1 and x_2; $\quad 8 - 24 + 14 - a = 0 \Rightarrow a = 2, b = 3$

42.(BCD) $2a = \sqrt{16+9} = 5$

$$C \equiv \left(0, \frac{5}{2}\right)$$



$$F_1 \equiv \left[2e, \frac{1}{2}(5-3e)\right],\ F_2 \equiv \left[-2e, \frac{1}{2}(5+3e)\right]$$

$$\frac{CF_1}{F_1A_1} = \frac{ae}{a-ae} = \frac{e}{1-e}$$

$$PF_1 + PF_2 = 2a$$

$$e^2 = \frac{5}{6} \Rightarrow e = \sqrt{\frac{5}{6}} \qquad \therefore b^2 = \frac{25}{24}$$

43.(AB) *a, b* and *c* are the roots of $8x^3+(\lambda+2)x^2-(2k+\lambda)x-27=0$ …………(i)

$\lambda^2+2\lambda(k+1)+4k=2^3.3^5$

$(\lambda+2)(\lambda+2k)=2^3.3^5$ ………(ii)

$$a+b+c=\frac{-(\lambda+2)}{8}$$

$$ab+bc+ca=\frac{-(2k+\lambda)}{8};\qquad abc=\frac{27}{8}$$

$$(a+b+c)(ab+bc+ca)=\left(\frac{-(\lambda+2)}{8}\right)\left(\frac{-(2k+\lambda)}{8}\right)=\frac{2^3.3^5}{8\times 8}=\frac{3^5}{8}$$

$$(abc)(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{3^5}{8};\qquad \frac{27}{8}(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{3^5}{8}$$

Hence, $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=9$

Now, using $A.M\geq H.M.$, in *a, b, c*, we get

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geq\frac{1}{9}$$

Hence, $a=b=c=\frac{3}{2}$; Triangle is equilateral

44. (ABC) Let *a* and *b* be the number of shots *A, B* respectively takes until they hit the target.

$$S=\sum_{n=1}^{\infty}(P(a=n))\times(P(b>n))$$

Now, for $P(a=n)$, A must miss their first $n-1$ shots and hit on the nth shot.

Thus, probability of $A=n=\left(1-\frac{3}{5}\right)^{n-1}\times\frac{3}{5}=\frac{3}{2}\times\left(\frac{2}{5}\right)^n$

Next we have $P(b>n)=\sum_{k=n+1}^{\infty}P(b=k)=\sum_{k=n+1}^{\infty}\left(\frac{2}{7}\right)^{k-1}\times\frac{5}{7}=\left(\frac{2}{7}\right)^n$

Thus, $S=\frac{3}{2}\sum_{k=n}^{\infty}\left(\frac{2}{5}\times\frac{2}{7}\right)^n=\frac{3}{2}\times\frac{\frac{4}{35}}{1-\frac{4}{35}}=\frac{6}{31}$ (A)

Now, probability that *A, B* require the same number of shots will be:

$$\sum_{k=n}^{\infty}(P(a=n))\times(P(b=n))=\sum_{n=1}^{\infty}\left(\left(\frac{4}{35}\right)^{n-1}\times\frac{3}{5}\times\frac{5}{7}\right)=\frac{3}{7}\times\frac{1}{1-\frac{4}{35}}=\frac{15}{31}\qquad\text{(C)}$$

Now, probability that *B* require less shots than *A* will be

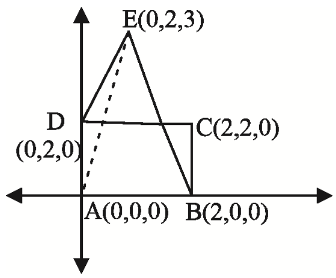
$1-\frac{16}{31}-\frac{15}{31}=\frac{10}{31}$ (B)

45.(AC) Plane ABE is

$$\begin{vmatrix} x & y & z \\ 0 & 2 & 3 \\ 2 & 0 & 0 \end{vmatrix} = 0$$

$$x(0) - y(-6) + z(-4) = 0$$

$$3y - 2z = 0$$



46.(BCD)

$3I = 5A^2 - A^3 \;\Rightarrow A^3 - 5A^2 + 3I = 0 \Rightarrow \lambda^3 - 5\lambda^2 + 3 = 0$ …(i)

Also, $|A - \lambda I| = 0 \;\Rightarrow\; \lambda^3 - (a+3)\lambda^2 + \lambda(3a - 2b - 3c + 1) + a(3c-2) + b - 2c + 1 = 0$ …(ii)

From (i) and (ii)

We get $a + 3 = 5 \Rightarrow\; a = 2$

$$2b + 3c = 7$$

$$b + 4c = 6$$

$$b = 2$$

$$c = 1$$

47.(ABD) Put $a = g'(1), b = g''(2)$

$f(x) = x^2 + ax + b$ ………..(i)

$$g(x) = cx^2 + x(2x + a) + 2$$

$$g(x) = (c+2)x^2 + ax + 2$$

$$g'(x) = 2(c+2)x + a$$

$$g'(1) = 2(c+2) + a = a$$

$$c = -2$$

$$g''(x) = 2(c+2)$$

$$g''(2) = 2(c+2) = b$$

$$b = 0$$

$$f(x) = x^2 + ax$$

$$g(x) = ax + 2$$

Now, $c = f(1)$

$$f(1) = 1 + a = c - 2 \Rightarrow a = -3$$

$$f(x) = x^2 - 3x, g(x) = 2 - 3x$$

48.(CD) It is not necessary that $f(x)$ is differentiable $x = 2$

49.(743) $(EM)^T = 20I$.

Take transpose on both sides

$EM = 20I$

$(E+M)^T = 17(E-M)^T$

$(E)^T + M^T = 17\left(E^T - M^T\right)$

$16(E)^T = 18\left(M^T\right)$

Take transpose on both sides

$16E = 18M$; $\quad 8E = 9M \Rightarrow 64E^2 = 81M^2$

$E^2 + M^2 = \left(\frac{81}{64}+1\right)M^2 = \left(\frac{145}{64}\right)M^2$; $\qquad 8EM = 9M^2 \Rightarrow M^2 = \frac{160}{9}I$

$E^2 + M^2 = \left(\frac{145}{64}\right)\left(\frac{160}{9}I\right) = \frac{725}{18}I \Rightarrow a+b = 743$

50.(9) Given, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \cos\alpha$

Volume of tetrahedron $= \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

Now, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} 1 & \cos\alpha & \cos\alpha \\ \cos\alpha & 1 & \cos\alpha \\ \cos\alpha & \cos\alpha & 1 \end{vmatrix}$; $\qquad 36V^2 = (1+2\cos\alpha)(1-\cos\alpha)^2$

$36\times\frac{1}{360} = (1+2\cos\alpha)(1-\cos\alpha)^2 = \frac{1}{10}$; $\qquad 3\cos^2\alpha - 2\cos^3\alpha = \frac{9}{10}$

51.(200) $S = \frac{2+6}{4^{100}} + \frac{2+2(6)}{4^{99}} + \frac{2+3(6)}{4^{98}} + \ldots\ldots + \frac{2+99(6)}{4^2} + \frac{2+99(6)}{4^2} + \frac{2+100(6)}{4}$

$= \frac{1}{4^{100}}\left(2\sum_{n=0}^{99} 4^n + 6\sum_{n=1}^{100} n.4^{n-1}\right)$

$= \frac{1}{4^{100}}\left(2.\frac{4^{100}-1}{4-1} + 6\left(\frac{1-100\left(4^{100}\right)}{1-4} + \frac{4\left(1-4^{99}\right)}{(1-4)^2}\right)\right) = 200$

52.(10) $f(-x) = f(x), g(x)g(-x) = 1$

$\int_0^a f(x)dx = 10$

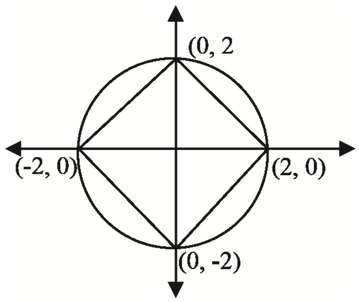
$I = \int_{-a}^{a} \frac{f(x)}{1+g(x)}dx$ ……….(i)

Using King's property $I = \int_{-a}^{a} \frac{f(-x)}{1+g(-x)}dx = \int_{-a}^{a} \frac{g(x)f(x)}{1+g(x)}dx$ …….(ii)

Equation (i), + equation (ii)

$2I = \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \Rightarrow I = \int_0^a f(x)\,dx = 10$

53.(8)



$A=\{z:|z|=2\}$ $\quad$ $A\cap B=\{|z^2-z\bar{z}|+|z^2+z\bar{z}|\le 4|z|\}$ using $z\bar{z}=4$

$A\cap B=\{z:|z-\bar{z}|+|z+\bar{z}|\le 4\}$

$A\cap B=\{z:|x|+|y|\le 2\}$

$\Rightarrow$ Area of the square is = 8 sq. units.

54.(17) $(y-2)^2=4(x-1)\equiv Y^2=4X$

$=P(at^2,2at)$

Tangent at P,

$tY=X+t^2$

$Q\equiv(-t^2,0),\ R\equiv(t^2,0)$

Area of ΔAQR, $\frac{1}{2}\times 2t^2\times t=\pm 64$

t $=\pm 4$

Now, abscissa of point P$\Rightarrow x-1=t^2=16$

$x=17$